Math 10B with Professor Stankova
Quiz 2; Tuesday, 1/30/2018
Section #211; Time: 11 AM
GSI name: Roy Zhao **Name:** ________________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. True **FALSE** There is only one way to prove a combinatorial identity such as $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$.

> **Solution:** We can prove it algebraically or combinatorially.

2. True **FALSE** When we are dealing with multiple pigeons and boxes, we can use pigeonhole principle to prove that every box must have at least one pigeon in it.

> **Solution:** Pigeonhole principle can only tell us about what happens in one box.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) For this problem, we want to line up 3 males and 10 females in a line.
   (a) (4 points) Prove that there must be at least 3 females that line up next to each other.

   > **Solution:** First place the males and then boxes be the spaces between them and on the sides. Therefore, there are 4 boxes. We put the 10 females in them and pigeonhole principle tells them that there must be a box with at least $10/4 = 2.5$ females, so a box with at least 3 females. This means that in this box, these 3 females are going to line up next to each other.

   (b) (2 points) Is it possible to line them up so that more than 3 females line up next to each other? If so, how?

   > **Solution:** We can have all of them line up next to each other like FFFFFFFFFFMMM.

   (c) (4 points) How many ways are there to line them up if we only care about the order of males and females (the males/females are indistinguishable).

Solution: There are $C(13, 10)$ ways to pick locations for the girls and order doesn't matter because the females are indistinguishable. Once we chose the location for the females, the location of the males is determined. Thus, the answer is just $\binom{13}{10}$.